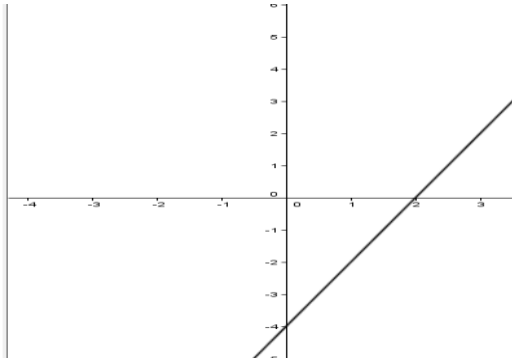


# Eerste Graadsfunctie $f(x) = ax + b$

EERSTE GRAADSFUNCTIE met  $a > 0 \rightarrow$  STIJGENDE RECHTE

$$f(x) = 2x - 4$$



$$f(x) = 2x - 4$$

$a = 2$ , dus stijgende rechte

Nulpunt :  $2x - 4 = 0$  of  $x = 2$ , dus  $(2, 0)$

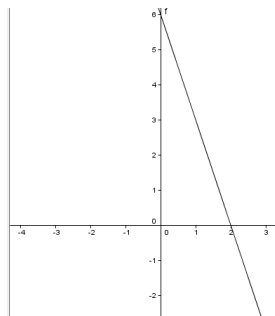
Snijpunt met Y-As =  $b = -4$ , dus  $(0, -4)$

Tekenverloop

|      |                |          |
|------|----------------|----------|
| x    | 0              | 2        |
| f(x) | ----- -4 ----- | 0 ++++++ |

EERSTE GRAADSFUNCTIE met  $a < 0 \rightarrow$  DALENDE RECHTE

$$-3x + 6$$



$$f(x) = -3x + 6$$

$a = -3$ , dus dalende rechte

Nulpunt :  $-3x + 6 = 0$  of  $x = 2$ , dus  $(2, 0)$

Snijpunt met Y-As =  $b = 6$ , dus  $(0, 6)$

Tekenverloop

|      |                  |         |
|------|------------------|---------|
| x    | 0                | 2       |
| f(x) | +++++++ 6 ++++++ | 0 ----- |

# Tweede Graadsfunctie $f(x) = ax^2 + bx + c$

**TWEEDE GRAADSFUNCTIE met  $a > 0$  = DALPARABOOL , 2 nulpunten**

$$f(x) = 2x^2 + 2x - 12$$

$a > 0$  , dus DALPARABOOL

Nulpunten :  $D = b^2 - 4ac = 4 - 4 \cdot 2 \cdot (-12) = 4 + 96 = 100 = 10^2 > 0$  , dus 2 nulpunten

$$x_1 = \frac{-b - \sqrt{D}}{2a} = \frac{-2 - 10}{4} = -3 \text{ of } (-3, 0) \text{ en } x_2 = \frac{-b + \sqrt{D}}{2a} = \frac{-2 + 10}{4} = 2 \text{ of } (2, 0)$$

$$\text{Ontbinden } f(x) = 2x^2 + 2x - 12 = 2(x-2)(x+3)$$

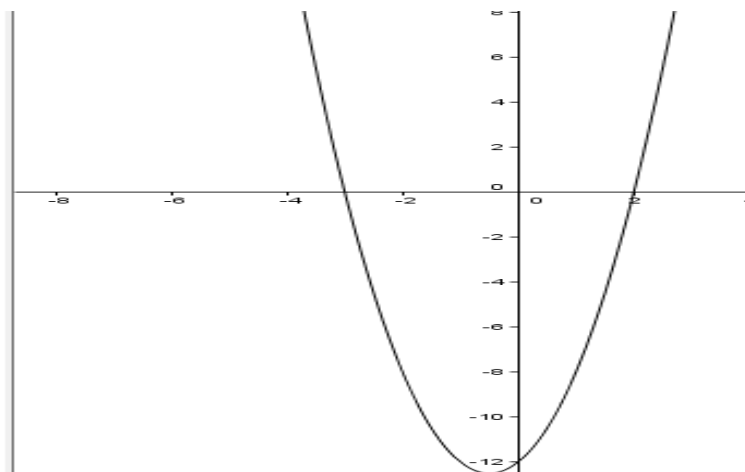
$$\text{Symmetrie as : } x = \frac{-b}{2a} = \frac{-2}{4} = -\frac{1}{2} \text{ ( of midden tussen beide nulpunten = -0,5)}$$

Snijpunt Y-as =  $b = -12$  , dus  $(0, -12)$

Functieverloop

|      |           |       |     |                |   |     |   |   |   |   |   |   |   |
|------|-----------|-------|-----|----------------|---|-----|---|---|---|---|---|---|---|
| x    | -3        | -0,5  | 0   | 2              |   |     |   |   |   |   |   |   |   |
| f(x) | +++++++ 0 | ----- | -12 | ----- 0 ++++++ |   |     |   |   |   |   |   |   |   |
|      | \         | \     | \   | \              | \ | MIN | / | / | / | / | / | / | / |

$$f(x) = 2x^2 + 2x - 12$$



**TWEEDE GRAADSFUNCTIE met  $a < 0$  = BERGPABOOL, 2 nulpunten**

$$f(x) = -2x^2 + 2x + 12$$

Nulpunten :  $D = b^2 - 4ac = 4 - 4 \cdot (-2) \cdot 12 = 4 + 96 = 100 = 10^2 > 0$  , dus 2 nulpunten

$$x_1 = \frac{-b - \sqrt{D}}{2a} = \frac{-2 - 10}{-4} = 3 \text{ of } (3,0) \text{ en } x_2 = \frac{-b + \sqrt{D}}{2a} = \frac{-2 + 10}{-4} = -2 \text{ of } (-2,0)$$

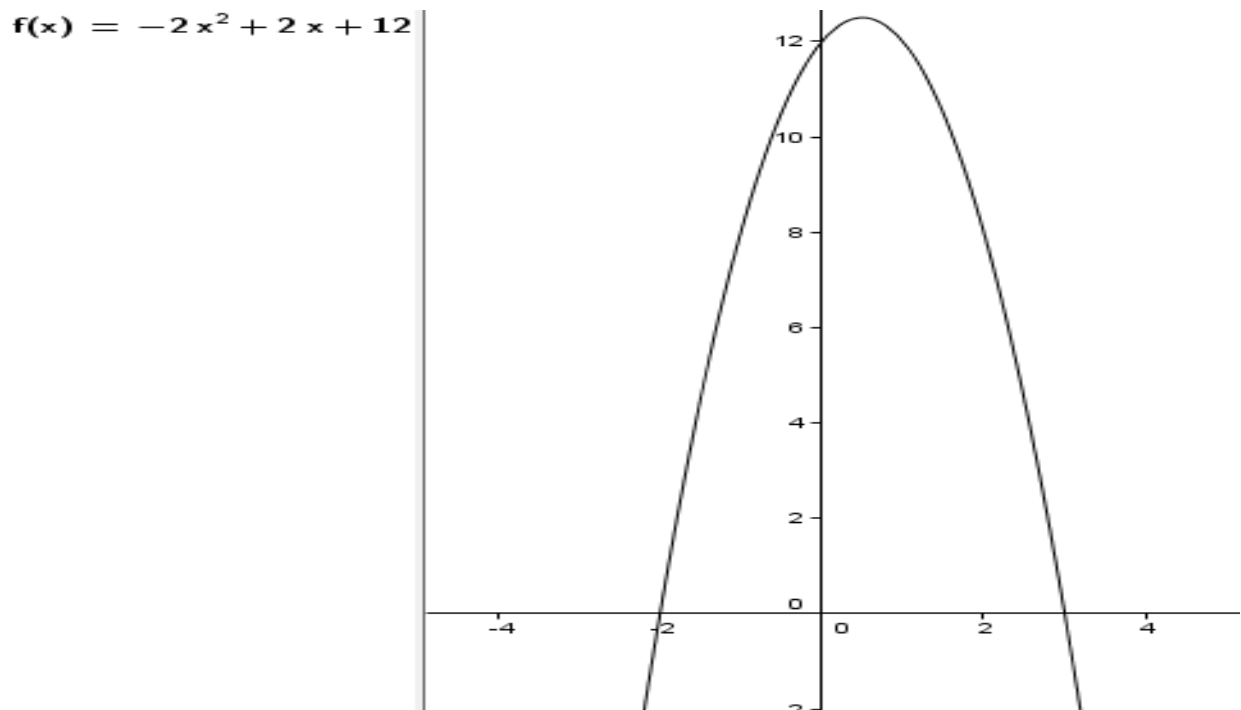
$$\text{Ontbinden } f(x) = -2x^2 + 2x + 12 = -2(x+2)(x-3)$$

$$\text{Symmetrie as : } x = \frac{-b}{2a} = \frac{2}{-4} = -\frac{1}{2} \text{ ( of midden tussen beide nulpunten = 0,5)}$$

$$\text{Snijpunt Y-as} = b = 12 \text{ , dus } (0,12)$$

Functieverloop

|      |           |               |            |       |
|------|-----------|---------------|------------|-------|
| x    | -2        | 0             | 0,5        | 3     |
| f(x) | -----     | +++++++ 12    | +++++++ -0 | ----- |
|      | / / / / / | MAX \ \ \ \ \ |            |       |



**TWEEDE GRAADSFUNCTIE met  $a > 0$  = DALPARABOOL , 1 dubbel nulpunt**

$$f(x) = 3x^2 - 6x + 3$$

Nulpunten :  $D = b^2 - 4ac = 36 - 4 \cdot 3 \cdot 3 = 36 - 36 = 0$  = dus 1 dubbel nulpunt

$$x_{1,2} = \frac{-b}{2a} = \frac{6}{6} = 1 \text{ of } (1,0)$$

Ontbinden  $f(x) = 3x^2 - 6x + 3 = 3(x-1)^2$

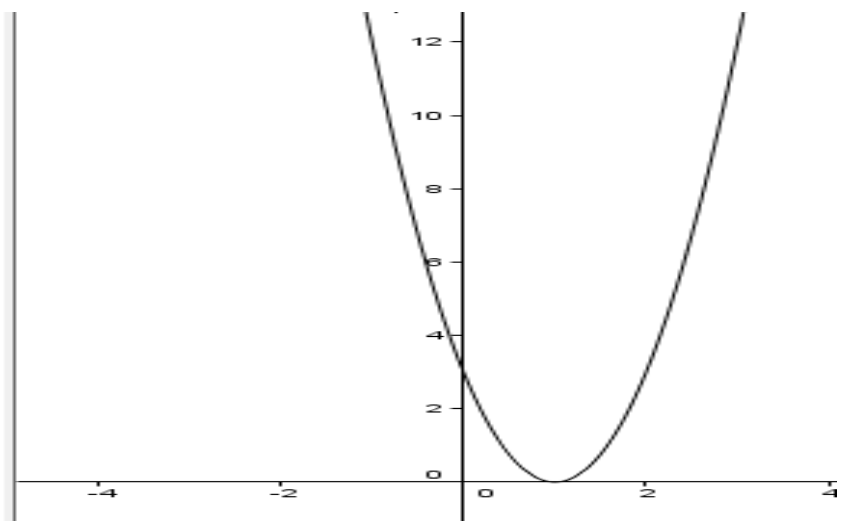
Symmetrie as :  $x = 1$  ( *waarde nulpunt* )

Snijpunt Y-as =  $b = 3$  , dus (0,3)

Functieverloop

|      |   |     |   |       |
|------|---|-----|---|-------|
| x    | 0 | 1   | 2 |       |
| f(x) | 3 | 0   | 3 | +++++ |
|      | \ | \   | \ | \     |
|      |   | MIN | / | /     |
|      |   |     | / | /     |
|      |   |     | / | /     |

$$f(x) = 3x^2 - 6x + 3$$



**TWEEDE GRAADSFUNCTIE met  $a < 0$  = BERGPABOOL , 1 dubbel nulpunt**

$$f(x) = -2x^2 + 8x - 8$$

Nulpunten :  $D = b^2 - 4ac = 64 - 4 \cdot (-2) \cdot (-8) = 64 - 64 = 0$  = dus 1 dubbel nulpunt

$$x_{1,2} = \frac{-b}{2a} = \frac{-8}{-4} = 2 \text{ of } (2,0)$$

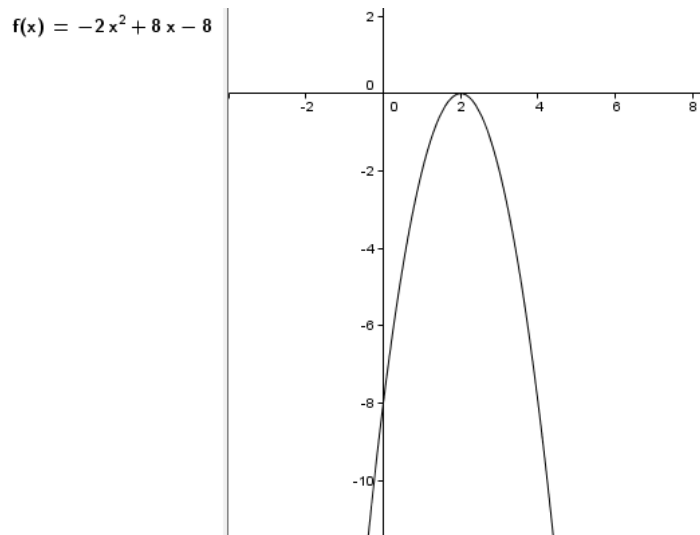
$$\text{Ontbinden } f(x) = -2x^2 + 8x - 8 = -2(x-2)^2$$

Symmetrie as :  $x = 2$  ( waarde nulpunt))

Snijpunt Y-as =  $b = -8$  , dus  $(0,-8)$

Functieverloop

|      |                |         |           |       |
|------|----------------|---------|-----------|-------|
| x    | 0              | 2       | 4         |       |
| f(x) | ----- -8 ----- | 0 ----- | -8 -----  | ----- |
|      | / / / / /      | / MAX \ | \ \ \ \ \ | \ \   |



**TWEEDE GRAADSFUNCTIE met  $a > 0$  DALPARABOOL , Geen nulpunt**

$$f(x) = x^2 + x + 1$$

Nulpunten :  $D = b^2 - 4ac = 1 - 4 \cdot 1 \cdot 1 = -3$  , dus geen nulpunt

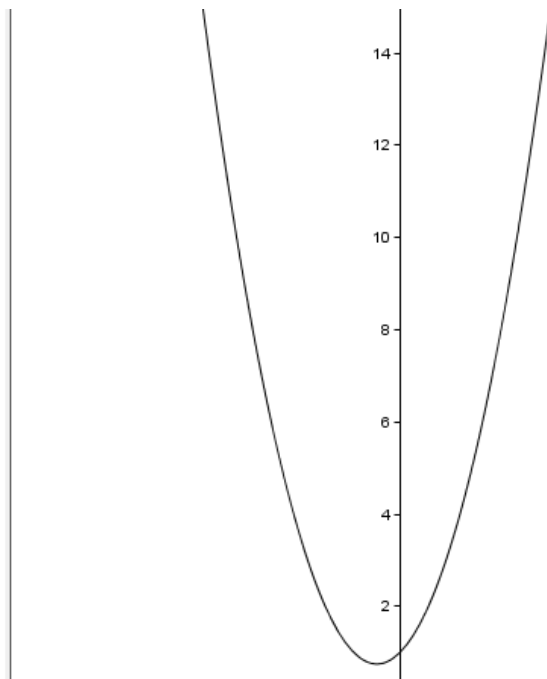
Symmetrie as :  $x = \frac{-b}{2a} = \frac{-1}{2}$

Snijpunt Y-as =  $b = 1$  , dus (0,1)

Functieverloop

|      |           |                   |           |
|------|-----------|-------------------|-----------|
| x    | -2        | -0,5              | 1         |
| f(x) | +++++++ 1 | +++++++ 1         | +++++++ 1 |
|      | \ \ \ \ \ | MIN / / / / / / / |           |

$$f(x) = x^2 + x + 1$$



**TWEEDE GRAADSFUNCTIE met  $a < 0$  BERGPANABOOL , geen nulpunt**

$$f(x) = -x^2 + x - 2$$

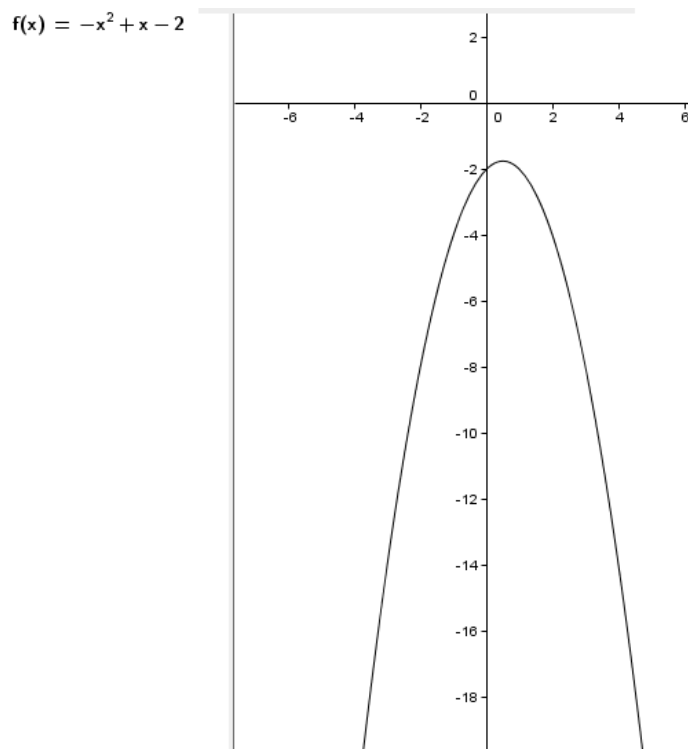
Nulpunten :  $D = b^2 - 4ac = 1 - 4 \cdot (-1) \cdot (-2) = -7$  , dus geen nulpunt

Symmetrie as :  $x = \frac{-b}{2a} = \frac{1}{2}$

Snijpunt Y-as =  $b = -2$  , dus  $(0,-2)$

## Functieverloop

|      |    |       |   |   |   |       |   |    |       |
|------|----|-------|---|---|---|-------|---|----|-------|
| x    | 0  | 0,5   | 1 |   |   |       |   |    |       |
|      |    |       |   |   |   |       |   |    |       |
| f(x) | -2 | ----- |   |   |   |       |   | -2 | ----- |
|      | /  | /     | / | / | / | MAX \ | \ | \  | \     |



# Derde Graadsfunctie $f(x) = ax^3 + bx^2 + cx + d$

DERDE GRAADSFUNCTIE met  $a > 0$  , 3 nulpunten

$$f(x) = 2x^3 - 12x^2 + 22x - 12$$

Nulpunten

Regel van Horner      2      -12      22      -12

                                 2      -10      12

                 1      2      -10      12      0 , dus 1<sup>ste</sup> nulpunt ( 1,0)

$2x^2 - 10x + 12$  geeft  $D = 100 - 96 = 4 = 2^2 > 0$  , dus 2 extra nulpunten

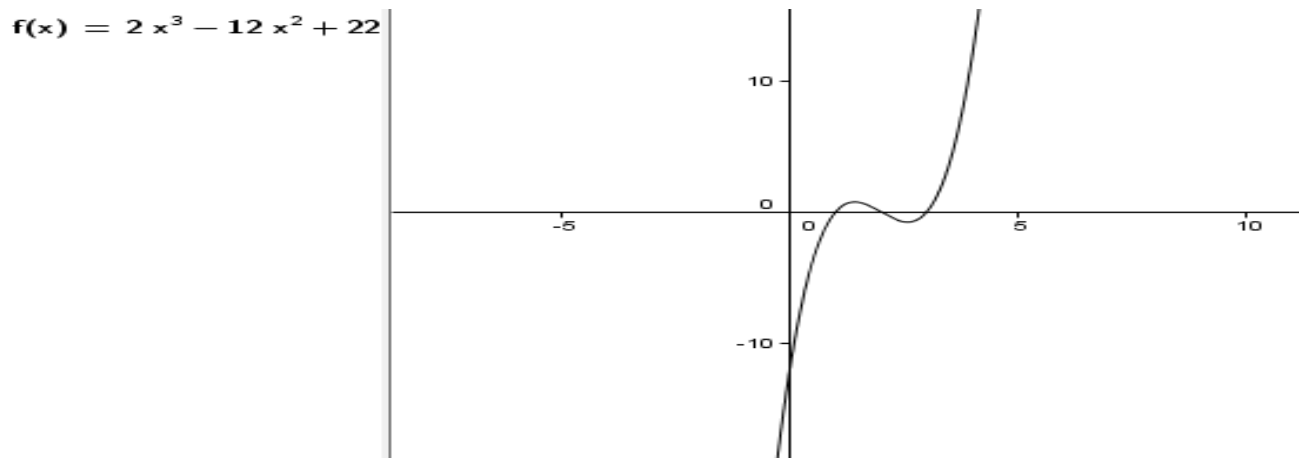
$(10+2)/4 = 3$  of (3,0) en  $(10-2)/4 = 2$  of (2,0)

Ontbinding  $f(x) = 2x^3 - 12x^2 + 22x - 12 = 2(x-1)(x-2)(x-3)$

Snijpunt Y-as =  $b = -12$  of (0,-12)

Functieverloop

|       |           |           |               |   |
|-------|-----------|-----------|---------------|---|
| x     | 0         | 1         | 2             | 3 |
| ----- |           |           |               |   |
| f(x)  | -12       | 0         | 0             | 0 |
|       | / / / / / | MAX \ \ \ | MIN / / / / / |   |



## DERDE GRAADSFUNCTIE met $a < 0$ , 3 nulpunten

$$f(x) = -x^3 + 7x + 6$$

Nulpunten

|                  |    |    |    |    |                                        |
|------------------|----|----|----|----|----------------------------------------|
| Regel van Horner | -1 | 0  | 7  | 6  |                                        |
|                  |    | 1  | -1 | -6 |                                        |
|                  | -1 | -1 | 1  | 6  | , dus 1 <sup>ste</sup> nulpunt ( -1,0) |

$$-x^2 + x + 6 \text{ geeft } D = 1 + 24 = 25 = 5^2 > 0, \text{ dus 2 extra nulpunten}$$

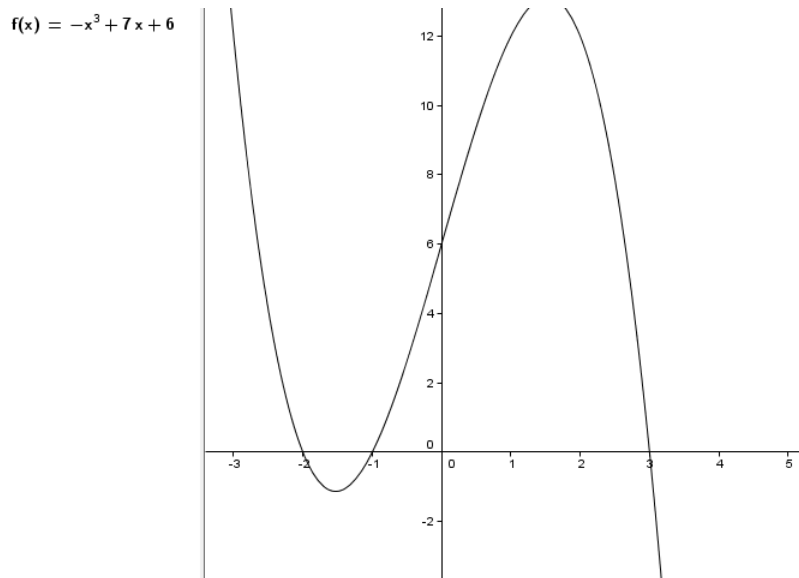
$$(-1+5)/-2 = -2 \text{ of } (-2,0) \text{ en } (-1-5)/-2 = 3 \text{ of } (3,0)$$

$$\text{Ontbinding } f(x) = -x^3 + 7x + 6 = -(x+1)(x+2)(x-3)$$

$$\text{Snijpunt Y-as } = b = 6 \text{ of } (0,6)$$

Functieverloop

|      |           |     |           |               |
|------|-----------|-----|-----------|---------------|
| x    | -2        | -1  | 0         | 3             |
|      |           |     |           |               |
| f(x) | +++++     | 0   | -----     | 0             |
|      | +++++     | 6   | +++++     | 0             |
|      | -----     | 0   | -----     | 0             |
|      | \ \ \ \ \ | MIN | / / / / / | MAX \ \ \ \ \ |



**DERDEGRAADSFUNCTIE met  $a > 0$  , 1 dubbel nulpunt gevolgd door 1 enkelvoudig nulpunt**

$$f(x) = x^3 - 3x - 2$$

Nulpunten

Regel van Horner            1        0       -3    -2

                                     -1       1       2

                         -1       1       -1       -2       0       , dus 1<sup>ste</sup> nulpunt ( -1,0)

$x^2 - x - 2$  geeft  $D = 1 + 8 = 9 = 3^2 > 0$  , dus 2 extra nulpunten

$(1+3)/2 = 2$  of (2,0) en  $(1-3)/2 = -1$  of (-1,0) , dus (-1,0) dubbel nulpunt en  $-1 < 2$

Ontbinding  $f(x) = x^3 - 3x - 2 = (x+1)^2(x-2)$

Snijpunt Y-as = b = -2 of (0,-2)

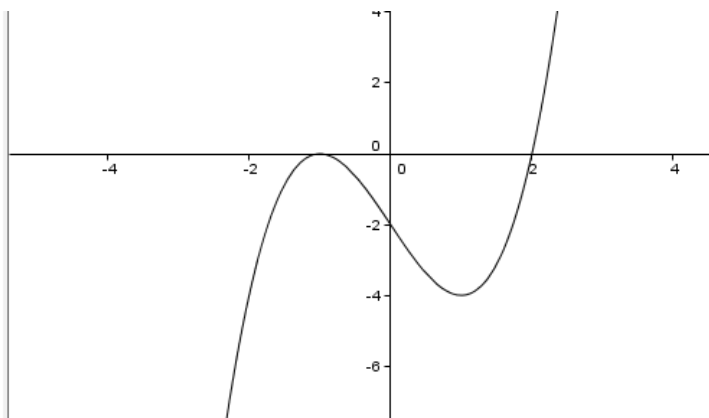
Functieverloop

x                                    -1                                    0                                    2

f(x ) ----- 0 ----- -2 ----- 0 ++++++

     /   /   /   /MAX\ \   \   \   \   \   \   MIN /   /   /   /   /

$$f(x) = x^3 - 3x - 2$$



**DERDE GRAADSFUNCTIE met  $a < 0$  , 1 dubbel nulpunt gevolgd door 1 enkelvoudig nulpunt**

$$f(x) = -x^3 - 3x^2 + 4$$

Nulpunten

Regel van Horner

|    |    |    |    |   |
|----|----|----|----|---|
| -1 | -3 | 0  | 4  |   |
|    | -1 | -4 | -4 |   |
| 1  | -1 | -4 | -4 | 0 |

, dus 1<sup>ste</sup> nulpunt ( 1,0)

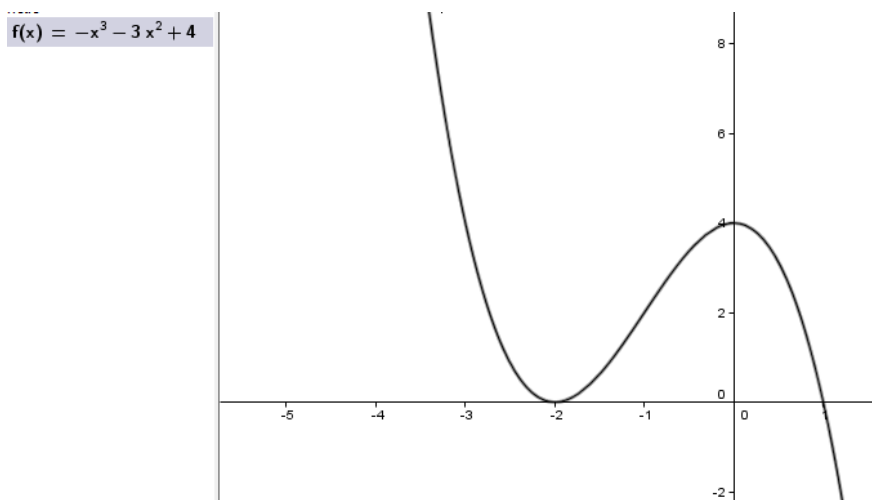
$-x^2 - 4x - 4$  is merkwaardig product  $-(x+2)^2$  dus dubbel nulpunt, met  $-2 > 1$

Ontbinding  $f(x) = -x^3 - 3x^2 + 4 = -(x+2)^2(x-1)$

Snijpunt Y  $-as = b = 4$  of (0,4)

Functieverloop

|      |                                 |         |         |
|------|---------------------------------|---------|---------|
| x    | -2                              | 0       | 1       |
|      |                                 |         |         |
| f(x) | +++++ 0                         | +++++ 4 | +++++ 0 |
|      | \ \ \ MIN / / / / MAX \ \ \ \ \ |         |         |



**DERDE GRAADSFUNCTIES met  $a > 0$  , 1 enkelvoudig nulpunt gevolgd door 1 dubbel nulpunt**

$$f(x) = x^3 - 3x^2 + 4$$

Nulpunten

|                  |   |    |   |    |
|------------------|---|----|---|----|
| Regel van Horner | 1 | -3 | 0 | 4  |
|                  |   | -1 | 4 | -4 |

|    |   |    |   |   |                                        |
|----|---|----|---|---|----------------------------------------|
| -1 | 1 | -4 | 4 | 0 | , dus 1 <sup>ste</sup> nulpunt ( -1,0) |
|----|---|----|---|---|----------------------------------------|

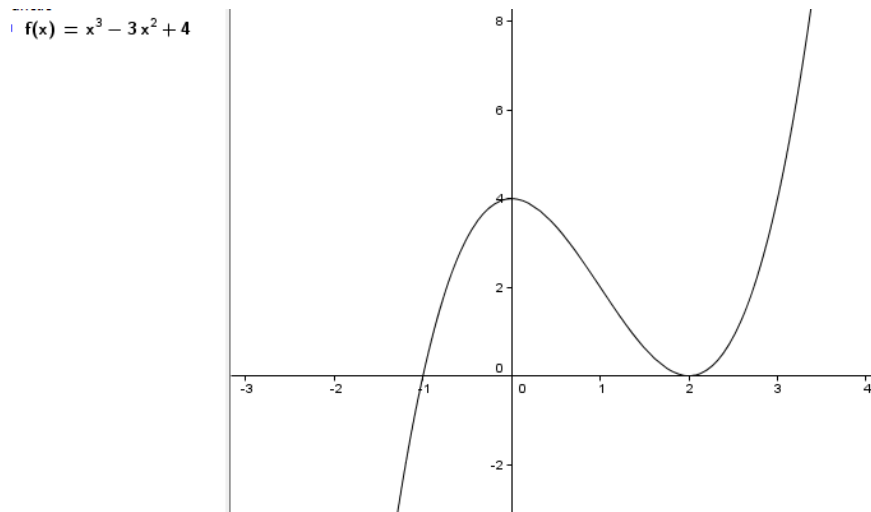
$x^2 - 4x + 4$  is merkwaardig product  $= (x-2)^2$  dus dubbel nulpunt met  $-1 < -2$

Ontbinding  $f(x) = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

Snijpunt Y –as = b = 4 of (0,4)

Functieverloop

|                                   |    |   |   |
|-----------------------------------|----|---|---|
| x                                 | -1 | 0 | 2 |
| -----                             |    |   |   |
| f(x)                              | 0  | 4 | 0 |
| +++++                             |    |   |   |
| / / / / / MAX \ \ \ \ MIN / / / / |    |   |   |



**DERDEGRAADSFUNCTIE met  $a < 0$  , 1 enkelvoudig nulpunt gevolgd door 1 dubbel nulpunt**

$$f(x) = -x^3 + 3x - 2$$

## Nulpunten

|                  |    |    |    |    |                                       |
|------------------|----|----|----|----|---------------------------------------|
| Regel van Horner | -1 | 0  | 3  | -2 |                                       |
|                  |    | -1 | -1 | 2  |                                       |
| 1                | -1 | -1 | 2  | 0  | , dus 1 <sup>ste</sup> nulpunt ( 1,0) |

$-x^2 - x + 2$  geeft  $D = 1 + 8 = 9 = 3^2 > 0$ , dus 2 extra nulpunten

$(1+3)/-2 = 2$  of  $(-2,0)$  en  $(1-3)/-2 = 1$  of  $(1,0)$  , dus  $(1,0)$  dubbel nulpunt en  $-2 < 1$

Ontbinding  $f(x) = -x^3 + 3x - 2 = -(x+2)(x-1)^2$

Snijpunt Y-as =  $b = -2$  of  $(0, -2)$

## Functieverloop

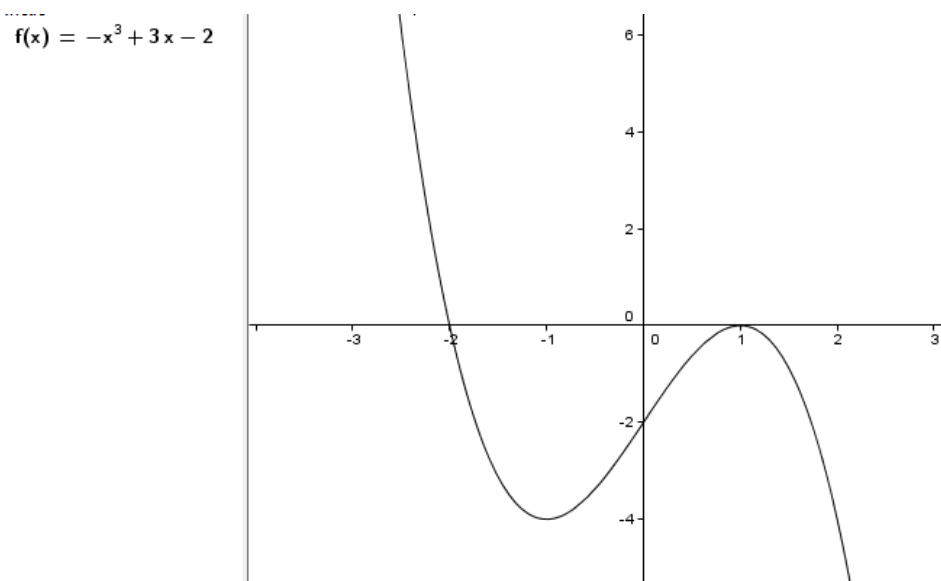
x

-2                      0                      1

-----

f(x) ++++++ 0 ----- -2 ----- 0 -----

         \ \ \ \ \ \ \ MIN / / / / / / MAX \ \ \ \ \



**DERDE GRAADSFUNCTIE met  $a > 0$  , 1 drievoudig nulpunt**

$$f(x) = x^3 - 3x^2 + 3x - 1$$

## Nulpunten

|                  |   |    |   |    |
|------------------|---|----|---|----|
| Regel van Horner | 1 | -3 | 3 | -1 |
|------------------|---|----|---|----|

$$\begin{matrix} 1 & -2 & 1 \end{matrix}$$

1      1      - 2      1      0      , dus 1<sup>ste</sup> nulpunt ( 1,0)

$x^2 - 2x + 1$  is een merkwaardig product  $= (x-1)^2$ , dus drievoudig nulpunt

Ontbinding  $f(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$

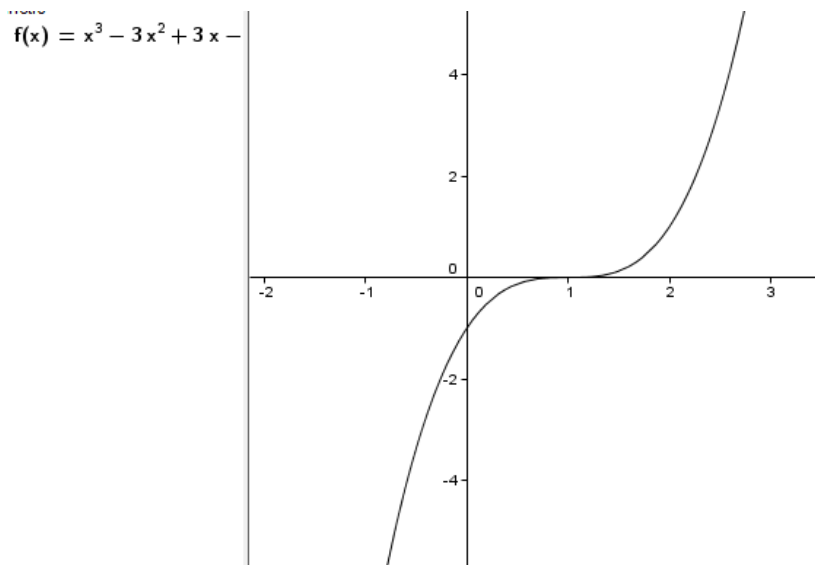
Snijpunt Y-as =  $b = -1$  of  $(0, -1)$

## Functieverloop

|   |   |   |
|---|---|---|
| x | 0 | 1 |
|---|---|---|

```
f(x) ----- -1 ----- 0 ++++++
```

/ / / / / / / / / / Buigpunt / / / / / / / /



# DERDE GRAADSFUNCTIE met $a < 0$ , 1 drievoudig nulpunt

$$f(x) = -x^3 - 6x^2 - 12x - 8$$

Nulpunten

Regel van Horner      -1      -6      -12      -8

                                 2      8      8

-2      -1      -4      -4      0      , dus 1<sup>ste</sup> nulpunt ( **-2,0**)

$-x^2 - 4x - 4$  is een merkwaardig product  $= -(x+2)^2$ , dus drievoudig nulpunt

Ontbinding  $f(x) = -x^3 - 6x^2 - 12x - 8 = -(x+2)^3$

Snijpunt Y  $-a = b = -8$  of (0,-8)

Functieverloop

x                                      -2                                      0

f(x)    ++++++ 0    ----- -8 -----

  \ \ \ buigpunt \ \ \ \ \ \ \ \ \ \ \

